FH-Kiel
University of Applied Sciences

© **Prof. Dr. Andreas Thiemer**
e-mail: andreas.thiemer@fh-kiel.de

Cobweb Models and Expectations

Summary:

The cobweb theorem is an old but beloved toy of economists to show market dynamics. Following Gandolfo (1997) the traditional cobweb model, where supply reacts to price with a lag of one period, can be considered as a particular case of a more general model involving price expectations. We examine the consequences of introducing different types of expectations.

The Static Market Model

Definitions :
p = price
x_S = supply
x_D = demand
p_E = equilibrium price
x_E = equilibrium quantity

Model equations:

$x_D(p) := a \cdot p + b$ (demand function)

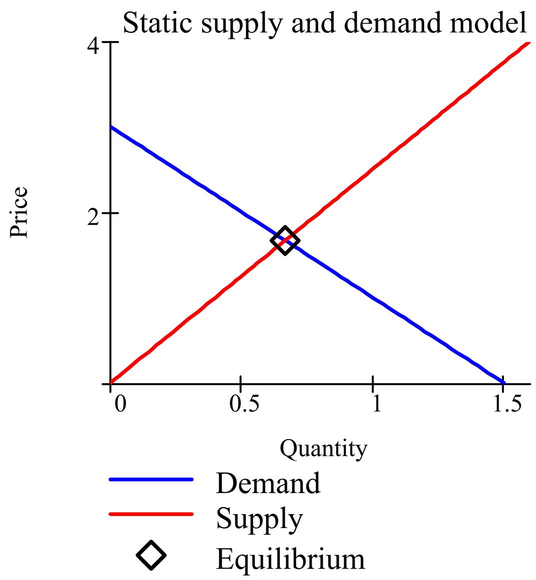
$x_S(p) := c \cdot p + d$ (supply function)

$x_S(p) = x_D(p)$ (equilibrium condition)

Solving this system yields the **equilibrium**:

$$p_E := \frac{b-d}{c-a} \qquad x_E := x_S(p_E)$$

Numerical example:



Parameters:

$a \equiv -0.5$

$b \equiv 1.5$

$c \equiv 0.4$

$d \equiv 0$

Solution:

$p_E = 1.667$

$x_E = 0.667$

Range of plotted functions: $pmax \equiv 4$

The General Dynamic Model with Price Expectations

$$x_{D_t} = a \cdot p_t + b \qquad \text{(demand function)}$$

$$x_{S_t} = c \cdot E(p_t) + d \qquad \text{(supply function)}$$

$$x_{D_t} = x_{S_t} \qquad \text{(equilibrium condition)}$$

$E(p_t)$ indicates the price expected by the producers for period t at the moment of starting their production in the beginning of this period. A first assumption on forming expectation is **perfect foresight**

$$E(p_t) = p_t = p_E$$

which is in accordance to **rational expectations** for the deterministic case. Then in any period t we get the above (long run) equilibrium solution. Other formations of price expectations are:

(I) Static or "naive" expectations

$$E(p_t) = p_{t-1}$$

(II) Normal price expectations

$$E_t(p_t) = p_{t-1} + \mu \cdot (p_N - p_{t-1}) \quad \text{with } 0<\mu<1 \text{ and a "normal" price } p_N$$

(III) Adaptive expectations

$$E(p_t) - E(p_{t-1}) = \beta \cdot (p_{t-1} - E(p_{t-1})) \quad \text{with } 0<\beta<1$$

(IV) Extrapolative ($\rho > 0$) and regressive ($\rho < 0$) expectations

$$E_t(p_t) = p_{t-1} + \rho \cdot (p_{t-1} - p_{t-2})$$

Now we analyze the cases (I) - (IV). The price in period 0 before starting production is $p_0 \equiv 0.4$. In the following examples the number of simulated periods is $Tmax \equiv 30$.

Case I: Static ("Naive") Expectations

The market clearing condition $x_{D_t} = x_{S_t}$ under the static price expectation hypothesis

$$E(p_t) = p_{t-1}$$

becomes:

$$a \cdot p_t - c \cdot p_{t-1} = d - b.$$

The **solution** of this first-order difference equation is:

$$p_t := (p_0 - p_E) \cdot \left(\frac{c}{a}\right)^t + p_E$$

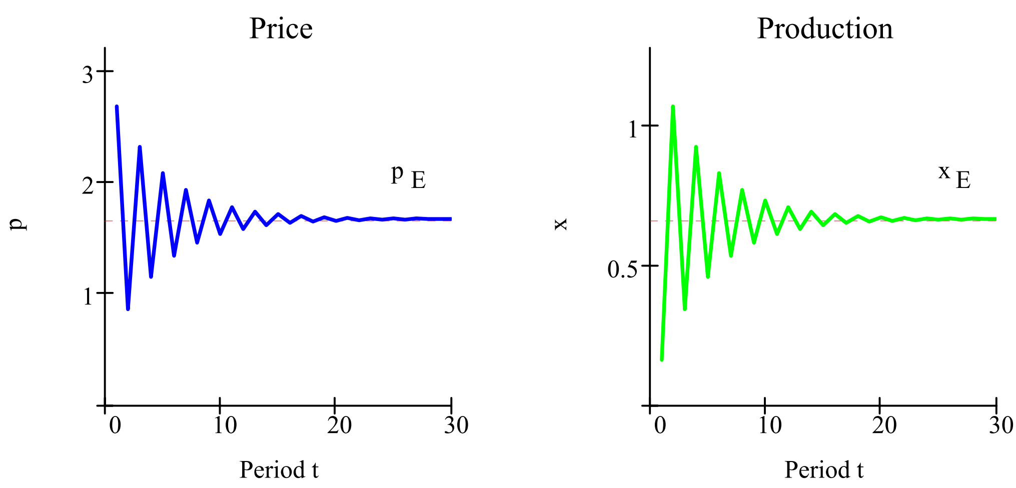
The **stability condition**, i.e. the condition that price converges to its equilibrium values, is:

$$\left|\frac{c}{a}\right| < 1$$

The **supply** (= production) in period t is a function of the price in t-1:

$$x_{S_t} := c \cdot p_{t-1} + d$$

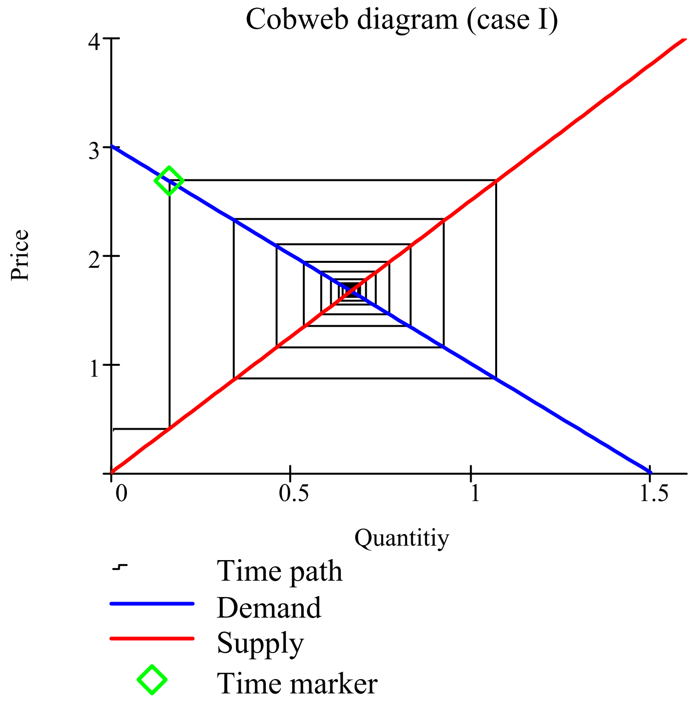
Now take a look at the oscillatory movement of price and production.



comment = "Stability condition is satisfied!"

Drawing these cycles into the demand-supply-figure yields the well known cobweblike path of price and production.

If you want to see where you are in period τ, set the "time marker" $\tau := 1$.



Case II: Normal-Price Expectations

The "normal" price p_N is the price which producers think will sooner or later obtain in the market. If the current price is different from p_N, they believe the former will modify, moving toward the latter:

$$E_t(p_t) = p_{t-1} + \mu \cdot (p_N - p_{t-1}) \text{ , with } 0<\mu<1$$

The reciprocal value of the parameter μ measures the expected speed of this process. (Note: $\mu = 0$ yields case I.) We assume $p_N = p_E$ (i.e. producers have perfect information but know that due to frictions, the price can't immediately go back to p_E when it is displaced from its long run equilibrium).

Substituting the expectation hypothesis into the equilibrium condition we obtain:

$$a \cdot p_t + b = c \cdot \left[p_{(t-1)} + \mu \cdot \left[p_E - p_{(t-1)} \right] \right] + d$$

The **solution** of this difference equation is (for $t := 1 .. Tmax$):

$$p_t := \left(p_0 - p_E\right) \cdot \left[\frac{c \cdot (1 - \mu)}{a} \right]^t + p_E$$

The **stability condition** is:

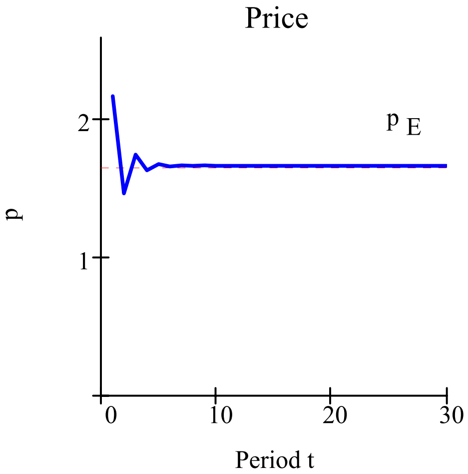
$$\left| \frac{c \cdot (1 - \mu)}{a} \right| < 1$$

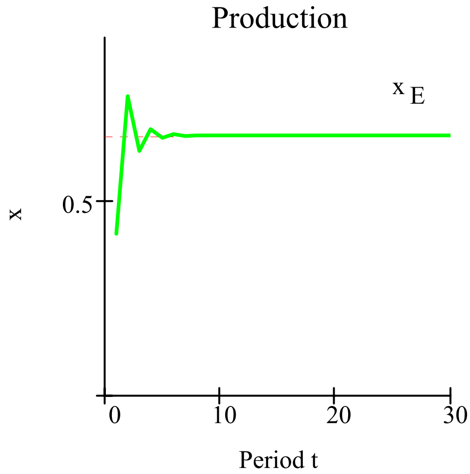
The **supply** in period t is a function of the price in t-1:

$$x_{S_t} := c \cdot \left[p_{t-1} + \mu \cdot \left(p_E - p_{t-1} \right) \right] + d$$

The introduction of expectations based on $p_N = p_E$ makes the model more stable compared to case I.

Choose parameter: $\mu \equiv 0.5$





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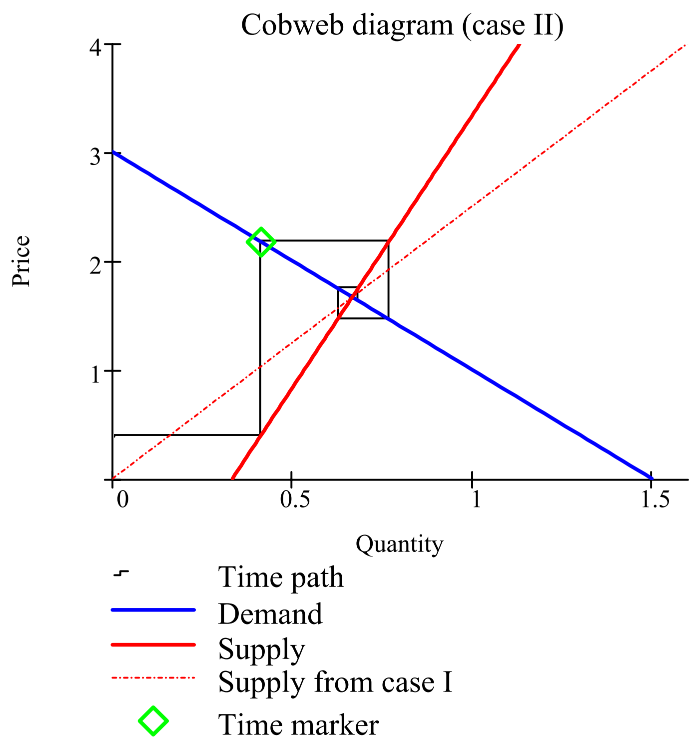
If you want to see where you are in period τ, set the "time marker" $\tau := 1$.



Case III: Adaptive Expectations

Expectations are revised ("adapted") in each period on the basis of the discrepancy between the observed value and the previously expected one, that is:

$$E\left(p_t\right) - E\left(p_{t-1}\right) = \beta\cdot\left(p_{t-1} - E\left(p_{t-1}\right)\right) \text{ with } 0<\beta<1$$

where β is the correction factor.

Substituting the expectation hypothesis into the equilibrium condition we obtain:

$$a\cdot p_t + b = (1-\beta)\cdot\left(b + a\cdot p_{t-1}\right) + \left(c\cdot\beta\cdot p_{t-1}\right) + \beta\cdot d$$

The **solution** of this difference equation is (with $t := 1 .. Tmax$):

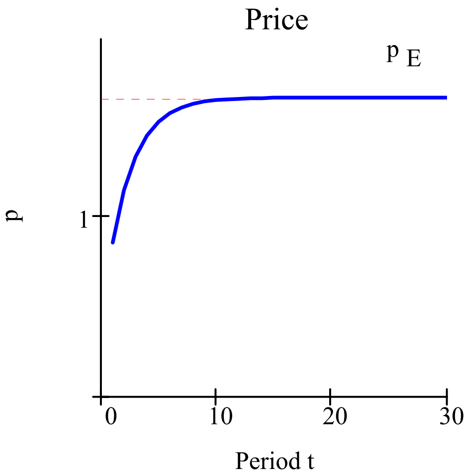
$$p_t := \left(p_0 - p_E\right)\cdot\left[\left(\frac{c}{a} - 1\right)\cdot\beta + 1\right]^t + p_E$$

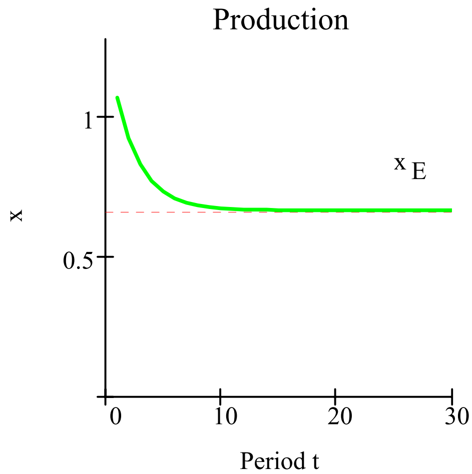
The **stability condition** is: $\left|\left(\frac{c}{a}-1\right)\cdot\beta+1\right|<1$

The **supply** in period t is a function of the price in t-1:

$x_{S_t} := ((1-\beta)\cdot a + c\cdot\beta)\cdot p_{t-1} + \beta\cdot d + (1-\beta)\cdot b$

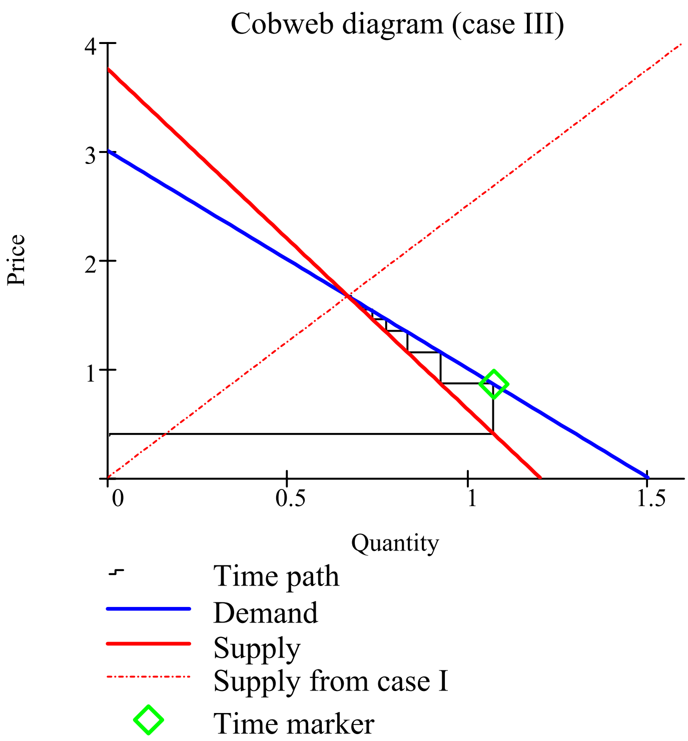
Choose parameter: $\beta \equiv 0.2$





comment = "Stability condition is satisfied!"

If you want to see where you are in period τ, set the "time marker" $\tau := 1$.



Case IV: Extrapolative and Regressive Expectations

Assume that expectations are formed according to the relation:

$$E_t(p_t) = p_{t-1} + \rho \cdot (p_{t-1} - p_{t-2})$$

If $\rho > 1$ we speak of extrapolative expectations. If $\rho < 0$ expectations are called regressive. $\rho = 0$ corresponds to case I (static expectations). After some substitutions into the market clearing condition we obtain:

$$a \cdot p_t - c \cdot (1+\rho) \cdot p_{t-1} + c \cdot \rho \cdot p_{t-2} = d - b$$

The **solution** of this second-order difference equation (for $t := 2..\,Tmax$ and $p_1 := p_0$) is:

$$p_t := \frac{(1+\rho) \cdot c}{a} \cdot p_{t-1} - \frac{\rho \cdot c}{a} \cdot p_{t-2} + \frac{d-b}{a}$$

The **stability conditions** are:

[1] $\dfrac{(1+\rho) \cdot c}{a} + \dfrac{\rho \cdot c}{a} > 0$

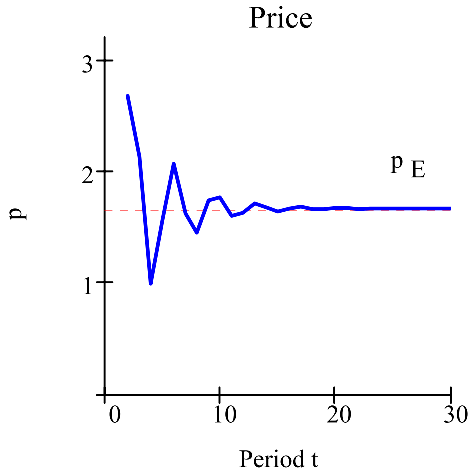
[2] $1 - \dfrac{\rho \cdot c}{a} > 0$

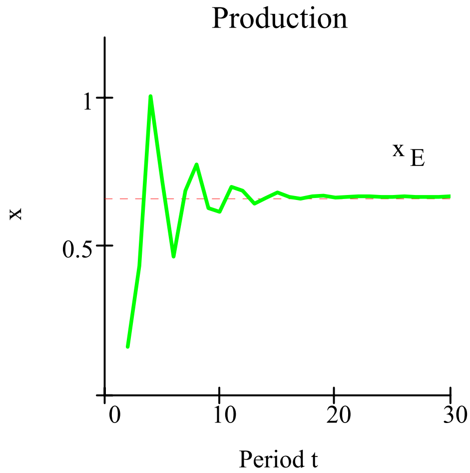
[3] $1 + \dfrac{(1+\rho) \cdot c}{a} + \dfrac{\rho \cdot c}{a} > 0$

Now the **supply** in period t depends not only from the price in period t-1 but also from p_{t-2}:

$$x_{S_t} := c \cdot \left[p_{t-1} + \rho \cdot (p_{t-1} - p_{t-2})\right] + d$$

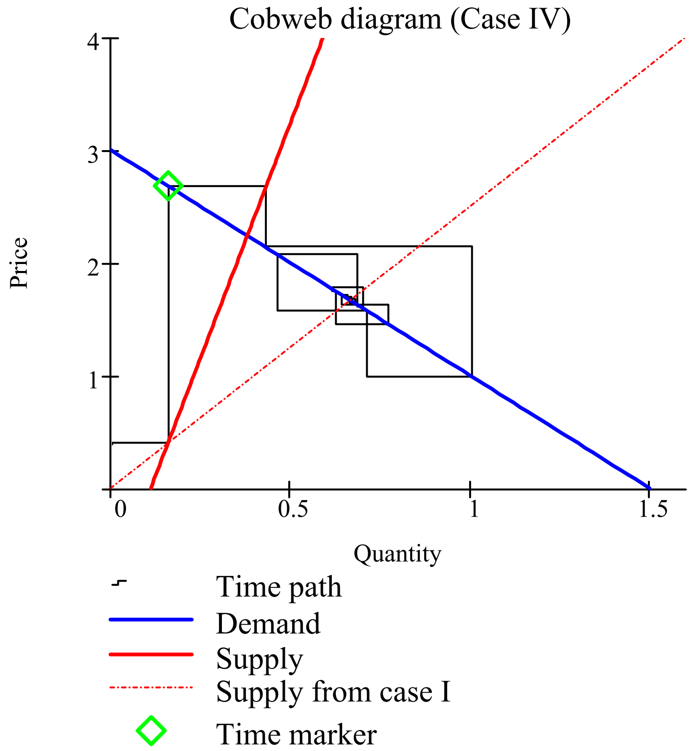
Choose parameter: $\rho \equiv -.7$





comment = "Stability conditions are satisfied!"

If you want to see where you are in period τ, set the "time marker" $\tau := 2$. *(Hint: The supply* x_{S_t} *is drawn as a function of* p_{t-1}. *Because the supply also depends on* p_{t-2} *the line changes its position in every period.)*



It's Your Turn!

Make ceteris paribus (small, sensible) changes to the parameter c. Note the threshold for which the models I - IV become unstable.

- Change the parameter γ in case II. Note its effect to the speed of the convergence process.

- Try different values of the correction factor β in case III (for instance: 0, 0.05, 0.5555, 0.75 and 1).

- Set different values for ρ in case IV (for instance: 0.2, 0.125, 0., -0.1, -0.25, -0.5, -0,75, -0.9, -1, -1.1 and -1.2).

Literature:

You will find all the important basics to these models (including the proofs of the stability conditions) in the excellent book:

Gandolfo, G.: Economic Dynamics. 3rd ed., Berlin/Heidelberg 1997.

Going back to the roots you should read:

Goodwin, R.M.: Dynamical Coupling with Especial Reference to Markets Having Production Lags. In: Econometrica, Vol. 15 (1947), p. 181 - 204.

Nerlove, M.: Adaptive Expectations and Cobweb Phenomena. In: Journal of Economics, Vol. 73 (1958), p. 227 - 240.