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Duopoly

Part I: The Solutions of Cournot and Stackelberg

Summary:

The charakteristic feature of oligopolistic markets is interdependence between firms. We restrict our example to a market with two firms supplying a homogenous good. In the Cournot model, each firm takes the quantity produced by its rival as given. An asymmetric kind of interdependence between firms is assumed in the Stackelberg model, in which one firm plays a leadership role and its rival merely follows. The "Stackelberg leader" strategically manipulates the quantity decision of its rival.

Basic Assumptions

Suppose the total (inverse) **market demand** curve for a homogenous good is given by:

$p(x) = \alpha - \beta \cdot x$ where $\alpha := 100$ $\beta := \frac{1}{2}$

There are two firms in the market producing this good with the **cost functions**:

$K_1(x_1) = a_1 \cdot x_1 + b_1 \cdot x_1^2$ where $a_1 := \frac{1}{10}$ $b_1 := \frac{1}{10}$

$K_2(x_2) = a_2 \cdot x_2 + b_2 \cdot x_2^2$ where $a_2 := \frac{1}{10}$ $b_2 := \frac{1}{10}$

The aggregated supply is $x_1 + x_2$.

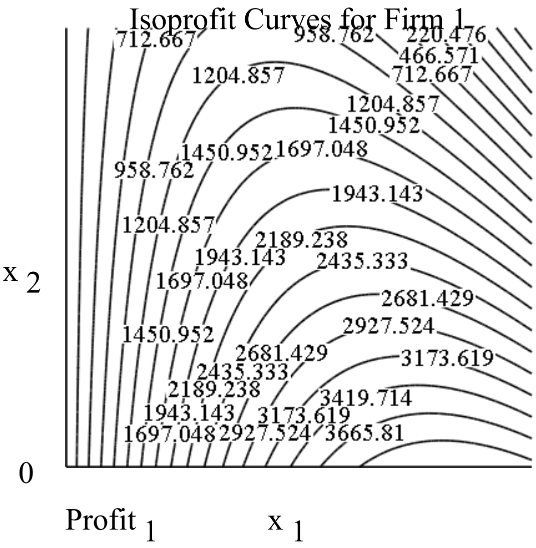
Case 1: Cournot-Duopoly

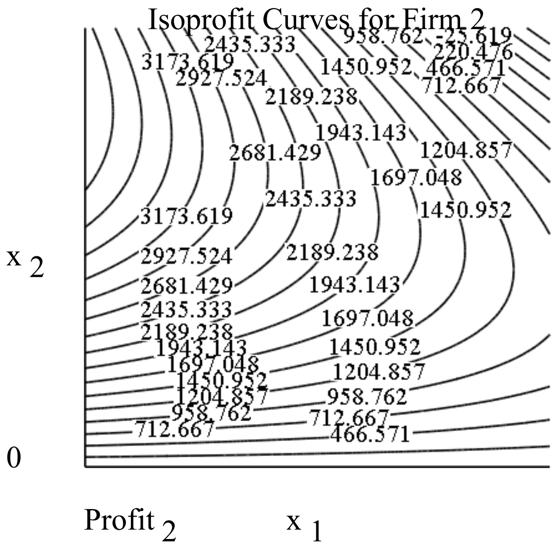
In the Cournot model each firm assumes that its rivals will continue producing at their current levels of output. Therefore, each duopolist treats the others quantity as a fixed number, one that will not respond to its own production decisions. But the profit Π_i of the firm i depends from the whole market supply:

$$\Pi_1(x_1, x_2) := \alpha \cdot x_1 - \beta \cdot (x_1 + x_2) \cdot x_1 - a_1 \cdot x_1 - b_1 \cdot x_1^2$$

$$\Pi_2(x_1, x_2) := \alpha \cdot x_2 - \beta \cdot (x_1 + x_2) \cdot x_2 - a_2 \cdot x_2 - b_2 \cdot x_2^2$$

The contour plot of the profit functions graphs the isoprofit lines. At any point on an isoprofit curve, the profit is the same.





The first order conditions of a profit maximum are

$$\frac{d}{dx_i}\Pi_i(x_1,x_2)=0 \quad \text{for} \quad i=1,2$$

Solving each of this equations to x_i, we get the reaction functions $X1(x_2)$ and $X2(x_1)$. Such a function tells how much output one firm will supply for each given amount produced by the other:

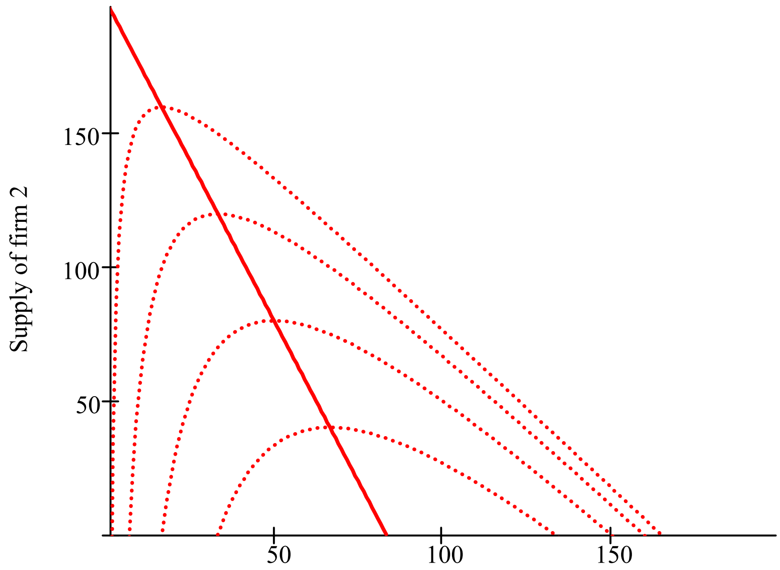
$$X1(x_2) := \left(\frac{d}{dx_1}\Pi_1(x_1,x_2)=0\right) \Bigg| \begin{array}{l}\text{auflösen}, x_1 \\ \text{gleit}, 5\end{array} \rightarrow 83.250 - .41667 \cdot x_2$$

$$X2(x_1) := \left(\frac{d}{dx_2}\Pi_2(x_1,x_2)=0\right) \Bigg| \begin{array}{l}\text{auflösen}, x_2 \\ \text{gleit}, 5\end{array} \rightarrow 83.250 - .41667 \cdot x_1$$

For example, the solution in the case of firm 1 as a monopolist is: $X1(0) = 83.25$

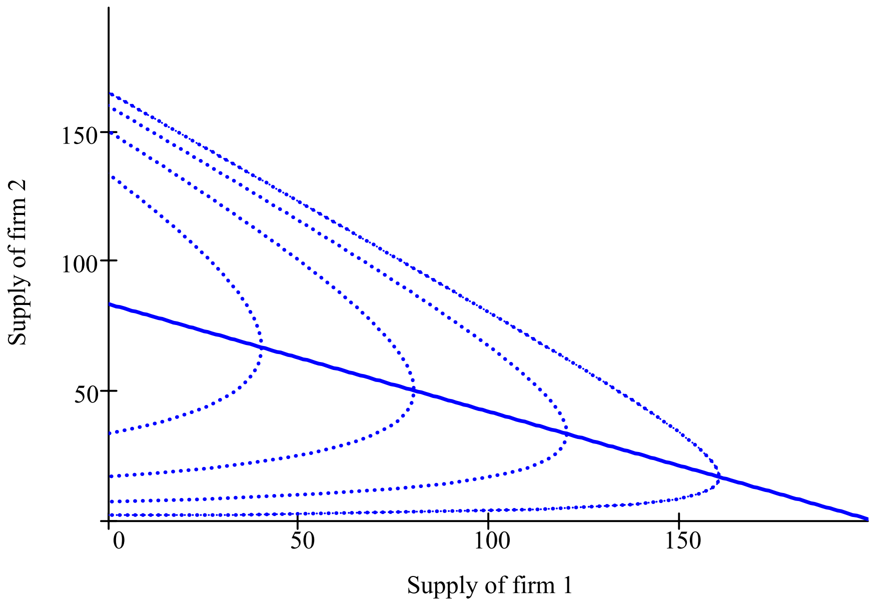
The next figures show the reaction functions for both firms together with some isoprofit curves.

Reaction Function for Firm 1



Supply of firm 1

Reaction Function for Firm 2



The duopolist are in a stable (Nash-)equilibrium at the point of intersection of their reaction functions. Let us compute this point $\left(X1_C, X2_C\right)$:

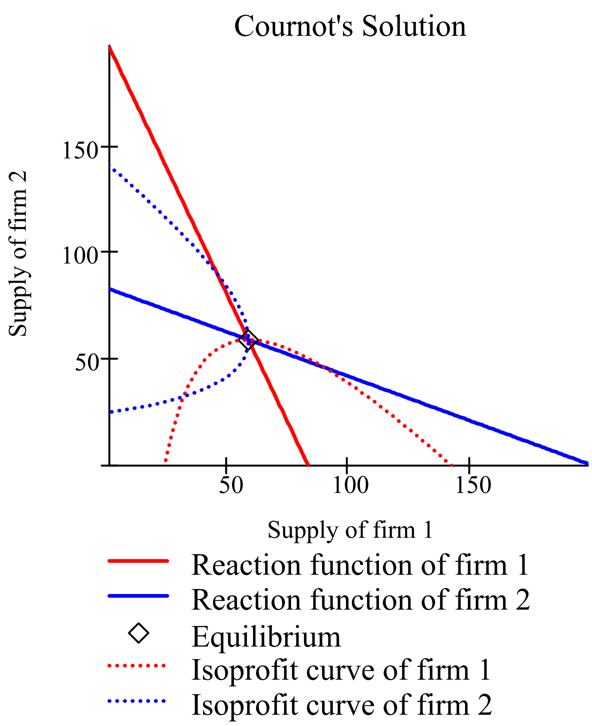
Initial guess values: $X1_C := 50$ $X2_C := 50$

Vorgabe

$$X1\left(X2_C\right) = X1_C$$

$$X2\left(X1_C\right) = X2_C$$

$$\begin{bmatrix} X1_C \\ X2_C \end{bmatrix} := \text{Suchen}\left(X1_C, X2_C\right)$$



Quantities:

$X1_C = 58.765$

$X2_C = 58.765$

Market price:

$p(X1_C + X2_C) = 41.235$

Profits:

$\Pi_1(X1_C, X2_C) = 2.072 \cdot 10^3$

$\Pi_2(X1_C, X2_C) = 2.072 \cdot 10^3$

Case 2: Stackelberg-Duopoly

Suppose firm 1 ("Stackelberg leader") knows that firm 2 ("Stackelberg follower") will treat firms 1's output level as given. Hence, by substituting $X2(x_1)$ for x_2 we can write the profit function of firm 1 as a function of x_1 alone:

$$\Pi_1(x_1) := \alpha \cdot x_1 - \beta \cdot (x_1 + X2(x_1)) \cdot x_1 - a_1 \cdot x_1 - b_1 \cdot x_1^2$$

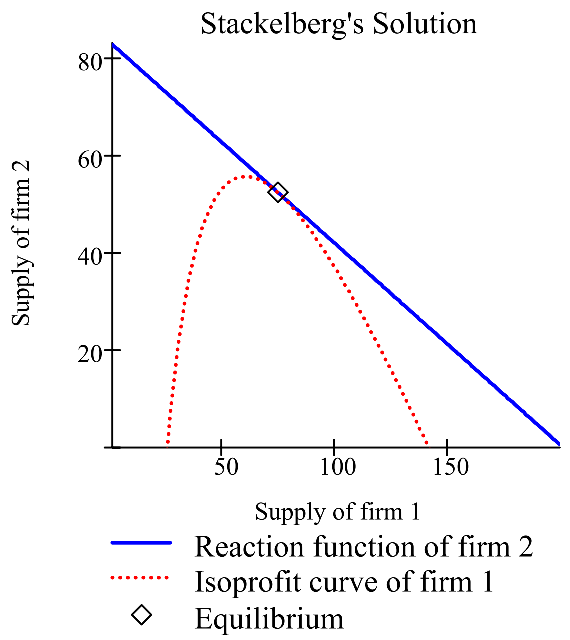
The first order condition of a profit maximum for firm 1 yields the **optimal supply** $X1_S$ **of the "Stackelberg leader"**:

$$X1_S := \left(\frac{d}{dx_1} \Pi_1(x_1) = 0 \right) \begin{array}{|l} \text{auflösen}, x_1 \\ \text{gleit}, 5 \end{array} \rightarrow 74.394$$

The **best response of the "Stackelberg follower"** is:

$$X2_S := X2(X1_S) \rightarrow 52.25225202$$

The point, where an isoprofit curve of the "Stackelberg leader" is tangent to the reaction function of the "Stackelberg follower", marks this equilibrium:



Market price:

$$p\left(X1_S + X2_S\right) = 36.677$$

Profits:

$$\Pi_1\left(X1_S\right) = 2.168 \cdot 10^3$$

$$\Pi_2\left(X1_S, X2_S\right) = 1.638 \cdot 10^3$$

It' s Your Turn!!

Solve exercise 14-1 and 14-3 from

R.H. Frank: Microeconomics and Behavior. New York 1994, Ch. 14

with the help of this worksheet.