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[redacted]

Endogenous Preference Cycles

Summary:

"Chaotic" consumption patterns over time may be a result of rational decisions. Benhabib and Day (1981) give an example for this, assuming a consumer with an endogenous Cobb-Douglas utility function, where the current preferences of this individual are influenced by the experience of the consumption history. This leads to the use of the logistic function, which is well known as a generator of chaos in discrete dynamical systems, opening the possibility of irregular preference cycles.

Static Utility Maximization

A consumer maximizes the **utility function**

$$U = x_1^{a} \cdot x_2^{1-a}$$

subject to the **budget constraint**

$$B \geq p_1 \cdot x_1 + p2 \cdot x_2,$$

where p_i are given prices and B is the given money income ("budget").

We find the optimum consumption choice, using the **Langrangian function**

$$L = x_1^{a} \cdot x_2^{1-a} + \lambda \cdot \left(B - p_1 \cdot x_1 - p2 \cdot x_2\right)$$

where λ is a shadow price - the marginal utility of income. Maximization of L demands the following necessary conditions:

$$\frac{d}{dx_1} L = a \cdot x_1^{(a-1)} \cdot x_2^{(1-a)} - \lambda \cdot p_1 = 0$$

$$\frac{d}{dx_2} L = (1-a) \cdot x_1^{a} \cdot x_2^{-a} - \lambda \cdot p_2 = 0$$

$$\frac{d}{d\lambda} L = B - p_1 \cdot x_1 - p2 \cdot x_2 = 0$$

These conditions can be solved to the quantities

$$X_1 = \frac{a \cdot B}{p_1} \qquad \text{and} \qquad X_2 = \frac{(1-a) \cdot B}{p_2}$$

where X_i stands for the **optimal consumption** of good i.

Introducing Endogenous Preferences

Assume that the utility weight (= a) is variable through time and depends on past optimal choices according to the function

$$a_{time} = \alpha \cdot X1_{time-1} \cdot X2_{time-1},$$

where α is a parameter that reflects the relevance of the consumption history for the current choice of the consumption quantities. From the results of the optimization procedure above we get:

$$a_{time} = \beta \cdot a_{time-1} \cdot \left(1 - a_{time-1}\right) \qquad \text{where} \qquad \beta := \frac{\alpha \cdot B^2}{p1 \cdot p2}$$

Now we introduce a simplification of our model assuming

$p1 \equiv 1$ $\quad$ $p2 \equiv 1$ $\quad$ $B \equiv 1$

such that $\alpha = \beta$. (You can relax this assumption later on!)

Exploring the Dynamics

Assuming an **initial value** to the **"utility weight parameter"** $a_0 := .1$

and given the **"experience parameter"** $\alpha \equiv 3.63$

we compute the optimal consumption and utility for $time := 1 .. T_{max}$ with $T_{max} \equiv 100$.

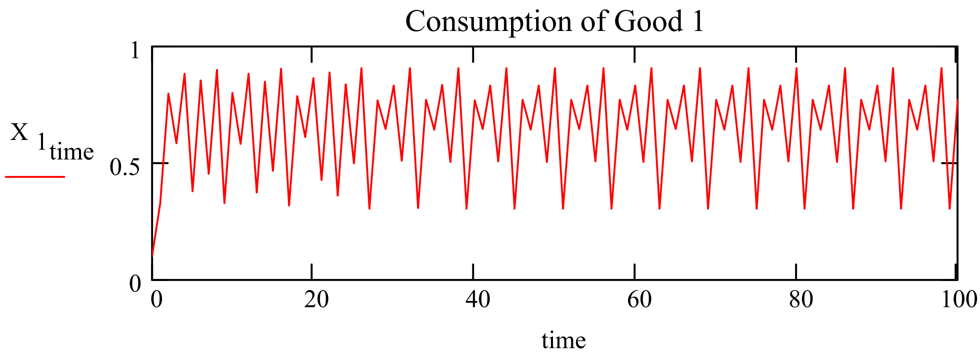
$$a_{time} := \beta \cdot a_{time-1} \cdot \left(1 - a_{time-1}\right)$$

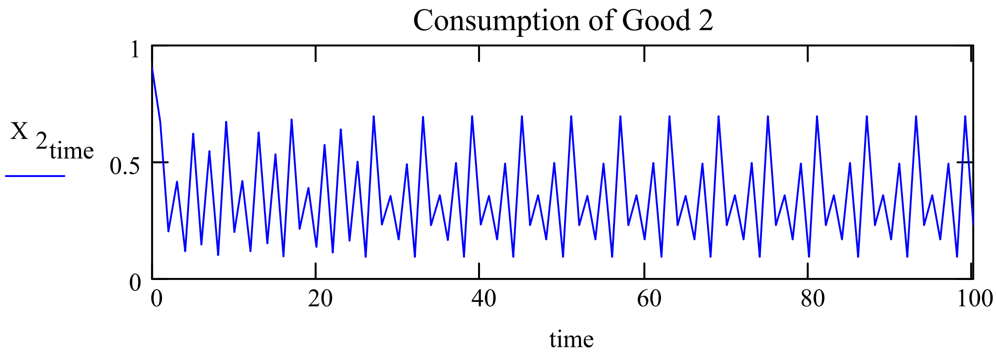
$$time := 0 .. T_{max}$$

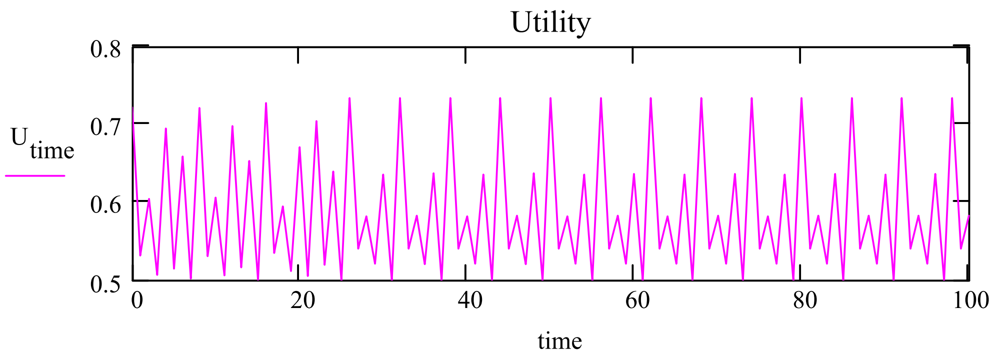
$$X1_{time} := \frac{B}{p1} \cdot a_{time}$$

$$X2_{time} := \frac{\left(1 - a_{time}\right) \cdot B}{p2}$$

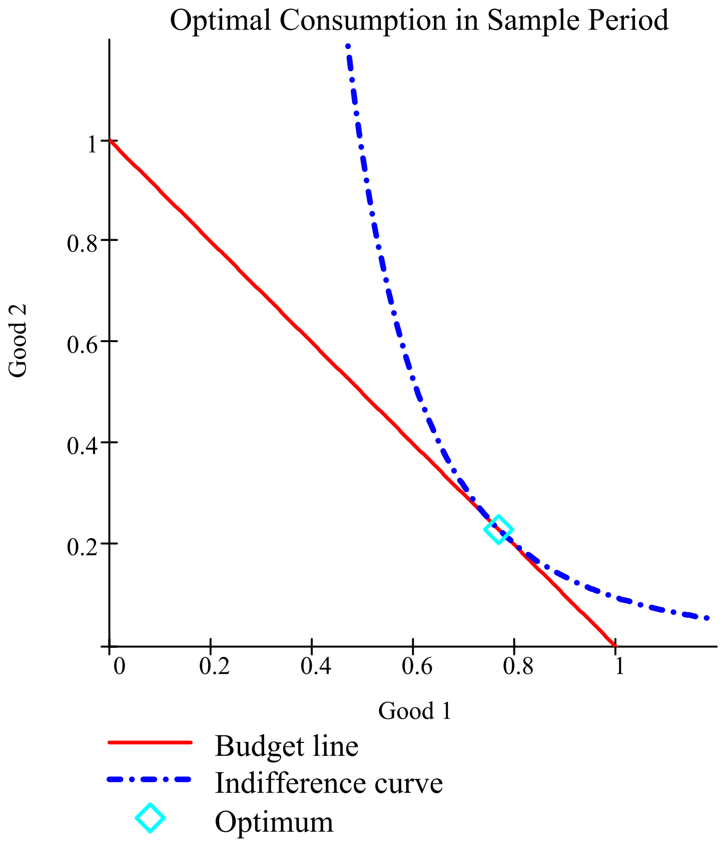
$$U_{time} := \left(X1_{time}\right)^{a_{time}} \cdot \left(X2_{time}\right)^{1 - a_{time}}$$


Consumption of Good 1
1
0.5
0
$X1_{time}$
0
20
40
60
80
100
time


Consumption of Good 2
1
0.5
0
$X2_{time}$
0
20
40
60
80
100
time


Utility
0.8
0.7
0.6
0.5
U_{time}
0
20
40
60
80
100
time

After selecting a sample period, the next figure sketches the optimal bundle of consumption goods:



Select sample period :

$\tau \equiv T_{max}$

with

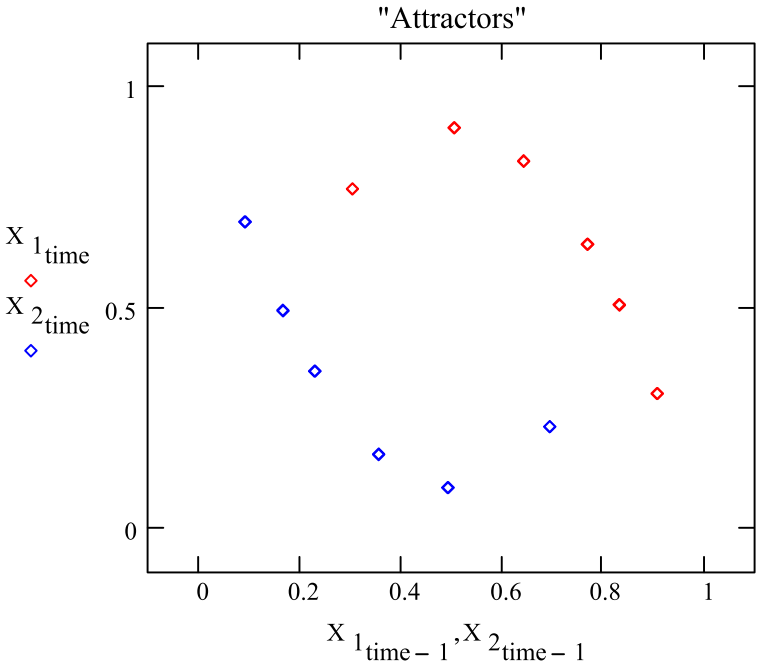
$X1_{\tau} = 0.77$

$X2_{\tau} = 0.23$

$U_{\tau} = 0.583$

The **phase diagram** plots actual consumption against consumption in the previous time period. If you cut off the first $T_{min} - 1$ periods given a high value of T_{max} (for example T_{max}= 10000), the remaining points show an "**attractor**" of the process. Especially you will identify cycles of different periodicity. In the case of "chaotic" movement for $\alpha = 4$ you will see, that the points form a parabola for every good. Therefore, the irregular sequence of consumption decisions doesn't follow an uniformly distributed stochastic pattern, as a first look to the time series for this case may erroneously suggest.

$T_{min} := \mathrm{trunc}\left(.75 \cdot T_{max}\right)$ $\qquad$ $time := T_{min} .. T_{max}$



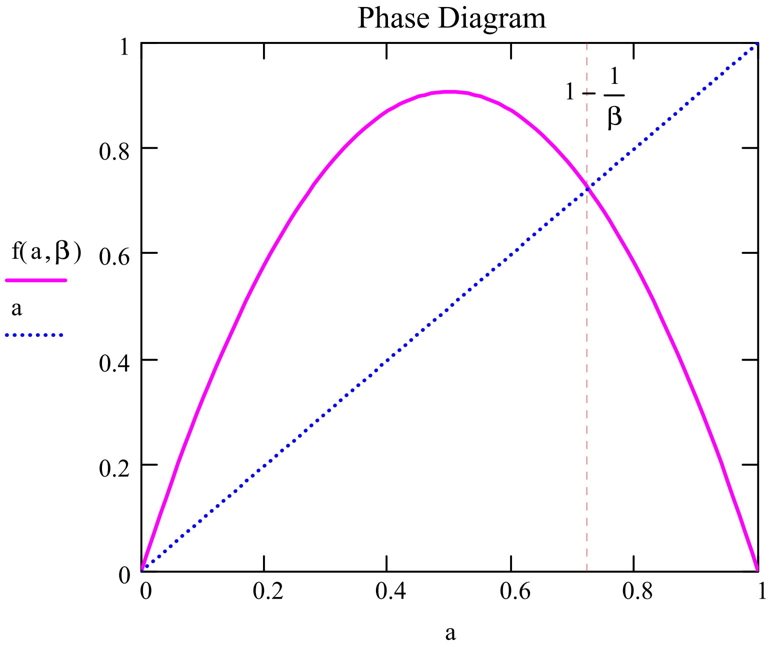
Stability Regions

The "utility weight parameter" a_{time} follows the well known **logistic function**. We redefine this function as an iterated map:

$$f(a,\beta) := \beta \cdot a \cdot (1 - a)$$

A **fixed point** is where $f(a,\beta) = a$ (this means $a_{time} = a_{time-1}$ for all periods). The blue dotted line in the figure below shows where this condition is fulfilled. The intersection point with the logistic curve identifies the fixed point. (Indeed there are two, including the trivial equilibrium a = 0).

$a := 0, .01 .. 1$



Positive fixed point:

$$1 - \frac{1}{\beta} = 0.725$$

with

$$\beta = 3.63$$

What about the stability of this fixed point?

Following for example Gandolfo (1997, pp. 506), the main types of behaviour can be classified according to the values of the parameter β.

Case I: $0<\beta<1$

The origin is stable while the second (irrelevant) negative fixed point is unstable.

Case II: $\beta=1$

The origin becomes the only (and stable) fixed point.

Case III: $1<\beta<3$

The origin becomes unstable, and a new positive fixed point in 1-1/β appears.

Case IV: $\beta=3$

A so called "flip bifurcation" occurs: The formerly stable equilibrium point becomes unstable and a new stable equilibrium state of period 2 emerges.

Case V: $3<\beta\le\beta_{\infty}=1+\sqrt{6}\approx 3.5699$

As β increases, an infinte sequence of flip bifurcations with period-doublings follows.

Case VI: $\beta_{\infty}<\beta\le 4$

In this "chaotic region" phenomena other than period doubling can be observed (cycles with odd periods, aperiodic oscillations etc.). But there remain also narrow subintervalls (called "windows") where low order periodic cycles (i.e., non-chaotic motion) prevail.

It's Your Turn!

- Now change α to explore the different kinds of dynamics that will occur. Try c.p. for example: α = 0.5, 2, 3, 3.3, 3.5, 3.55, 3.5699, 3.63, 3.7, 4.
- Try also different initial values of a_0 given α, and compare how sensitive the time path of consumption reacts.

Literature:

Benhabib, J./Day, R.: Rational Choice and Erratic Behaviour. Review of Economic Studies, Vol. 48 (1981), 459 - 471.

Gandolfo, G.: Economic Dynamics. 3rd ed., Berlin et al 1997.